# Electromagnetics of Superconductors

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*Abstract* —**The purpose of studying the electromagnetic behavior of superconductors is to identify the relevant material parameters of superconductive media and to examine their effects in the solution of classical electromagnetic boundary value problems. It is shown that a superconductor cannot be simply treated as a low loss conductor; rather, it should be treated as a negative dielectric material (with a negative dielectric constant). This approach is good only for vanishingly small field application with frequency significantly smaller than gap frequency, $f_c$, and temperature not too close to the critical temperature, $T_c$, of the superconductor. The electromagnetics of negative dielectric materials are discussed in terms of causality, perturbation technique, surface impedance, time-domain interpretation of current components, and computational electrodynamics.**

## I. Introduction

THE recent discovery of high-$T_c$ superconductors has fundamentally changed the prospects for applications of superconductive electronics and has generated considerable effort to apply these materials in a number of areas [1]–[3]. Computer simulation is needed to analyze and design superconductor components, devices, and circuits, especially for high frequencies. To pursue this goal, the electromagnetics of a superconducting medium must be clearly understood.

The objectives of this paper are to pave the way to solving superconductive boundary value problems and to find some possible new applications of superconductors through a better understanding of these materials from an electromagnetic point of view. The discussions presented in this paper do not seek to explore the physics of superconductors, but rather to use existing physical models to place superconductive material in its rightful position in macroscopic electrodynamics.

In classical electromagnetics, a material medium can be classified as either a conductor or a dielectric. In reexamining the London constitutive equations for superconductors, it is shown that a superconductor can be better described as a negative effective dielectric medium, i.e., the dielectric parameter having a negative real part [4], [5]. The implications of such a classification are discussed in the following sections with regard to causality, perturbation technique, surface waves, and computational electromagnetics.

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## II. Material Models of Superconductors

The material parameters of superconductors can be derived from the Mattis–Bardeen formula based on the microscopic Bardeen–Cooper–Schrieffer theory or a classical two-fluid model [6]–[10]. The Mattis–Bardeen formula predicts the sudden increase of loss at or above the gap frequency [6], but the two-fluid model does not. Despite its failure at the gap frequency, $f_c$, and at temperatures close to the critical temperature, $T_c$, the two-fluid model provides reasonable material parameters at frequencies significantly lower than the gap frequency. In fact, it is believed also to be a good approximate model for high-$T_c$ superconductors. On the other hand, the applicability of the Mattis–Bardeen formula to high-$T_c$ superconductor is debatable, because it only describes the extreme anomalous limit where the coherence length, $\xi$, is large compared with the penetration depth, $\lambda$. This limit is not realized, for example, in the YBaCuO ceramic superconductor. Measurements show that a typical value of the penetration depth $\lambda(0) = 1500$ Å for current flow within the copper–oxygen planes but reveal a considerably larger value for current flow perpendicular to the planes. Estimates of the coherence length give $\xi \approx 5$–$20$ Å, depending on the crystal orientation [11]. In this paper, we shall adopt the two-fluid model as the basis for the ensuing discussions, focusing on the macroscopic features of superconductors. In the discussions, we assume a low-field situation where the current density and magnetic field are much smaller than their critical values, so that we do not have to use the Ginzburg–Landau equations to characterize the superconductor.

### A. The Two-Fluid Model

In the two-fluid model, one postulates that a fraction of the conduction electrons is in the lowest-energy, or superconducting, state, with the remainder in the excited, or normal, state, with $n_s$ and $n_n$ being the superparticle and the normal-particle densities, respectively. Under the influence of external electric fields, the motion of normal electrons includes the effects of both resistance and inertia. The movement of superconducting electrons or, more appropriately, superconducting pairs is inertial only. This phenomenological model was originally used by London to explain the first microwave experiment with superconductors and it is still the framework in which most physicists and engineers visualize many superconductive phenomena [12]. This model is satisfactory for microwave engineers who wish to develop a working understanding and an intuitive feeling for the subject without having to go deeply into the details of complicated theories. The hydrodynamic equations of the paired and normal electrons can be written as follows:

$$m\frac{d\vec{v}_s}{dt} = -e\vec{E} \tag{1}$$

and

$$m\frac{d\langle\vec{v}_n\rangle}{dt} + m\frac{\langle\vec{v}_n\rangle}{\tau_n} = -e\vec{E} \tag{2}$$

where $\vec{v}_s$ is the velocity of the electron pairs, $\langle\vec{v}_n\rangle$ is the average velocity of the normal electrons, which have an average momentum relaxation time $\tau_n$, and $m$ and $e$ are the mass and charge of a single electron, respectively. Electron pairs are treated as collisionless particles, while the scattering process of normal electrons is approximately represented by the parameter $\tau_n$.

The total current density is the sum of two parts,

$$\vec{J} = \vec{J}_s + \vec{J}_n \tag{3}$$

with

$$\vec{J}_s = -n_s e\vec{v}_s \quad \text{and} \quad \vec{J}_n = -n_n e\langle\vec{v}_n\rangle \tag{4}$$

where $\vec{J}_s$ and $\vec{J}_n$ are the super and normal currents, respectively; $n_s$ and $n_n$ are the previously mentioned number densities of electron pairs and normal particles, respectively, and the summation of these two electron densities is a constant:

$$n = n_n + n_s. \tag{5}$$

The temperature dependence of $n_s$ and $n_n$ can be closely approximated by the Gorter–Casimir expressions:

$$\frac{n_s}{n} = 1 - \left(\frac{T}{T_c}\right)^4 \qquad \frac{n_n}{n} = \left(\frac{T}{T_c}\right)^4. \tag{6}$$

By defining the relative dielectric constant, $\epsilon_r$, as follows:

$$\nabla \times \vec{H} = j\omega\epsilon_0\epsilon_r\vec{E} = j\omega\epsilon_0\vec{E} + \vec{J} \tag{7}$$

where $\vec{J} = -n_s e\vec{v}_s - n_n e\langle\vec{v}_n\rangle$, one obtains the dielectric constant:

$$\epsilon_r(\omega) = 1 - \frac{\omega_s^2}{\omega^2} + \frac{\omega_n^2\tau_n}{j\omega(j\omega\tau_n + 1)} \tag{8}$$

where

$$\omega_s^2 = \frac{e^2 n_s}{m\epsilon_0} \qquad \omega_n^2 = \frac{e^2 n_n}{m\epsilon_0} \tag{9}$$

are the plasma frequencies of superparticles and normal particles. An alternative form for (8) in terms of the real and imaginary parts of $\epsilon_r(\omega)$ is

$$\epsilon_r(\omega) = \left(1 - \frac{\omega_s^2}{\omega^2} - \frac{\omega_n^2\tau_n^2}{\omega^2\tau_n^2 + 1}\right) - j\frac{\omega_n^2\tau_n}{\omega(\omega^2\tau_n^2 + 1)}. \tag{10}$$

## B. Causality

For the medium to be causal, the real and imaginary parts of $\epsilon_r(\omega)$ must be related by the Kramers–Kronig relation. Let $\epsilon_r(\omega) = \epsilon'(\omega) - j\epsilon''(\omega)$; then

$$\epsilon'(\omega) = 1 + \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\epsilon''(\xi)}{\xi - \omega}d\xi \tag{11}$$

$$\epsilon''(\omega) = -\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\epsilon'(\xi) - 1}{\xi - \omega}d\xi \tag{12}$$

where $P$ represents the principal value of the integral. It is readily shown that the dielectric parameter expressed in (10) does not satisfy (11) and (12), as one would have expected. This is because the superfluid makes no contribution to the imaginary part of $\epsilon_r(\omega)$ and a zero imaginary part in (11) cannot produce a real part other than unity. This situation can be remedied by considering the superfluid to be the limiting case of a lossy medium, i.e.,

$$\epsilon_r(\omega) = \lim_{\tau_s \to \infty}\left[\left(1 - \frac{\omega_s^2\tau_s^2}{\omega^2\tau_s^2 + 1} - \frac{\omega_n^2\tau_n^2}{\omega^2\tau_n^2 + 1}\right) - j\left(\frac{\omega_s^2\tau_s}{\omega(\omega^2\tau_s^2 + 1)} + \frac{\omega_n^2\tau_n}{\omega(\omega^2\tau_n^2 + 1)}\right)\right]. \tag{13}$$

This limiting process is similar to many other problems in classical electrodynamics where the medium has to be considered slightly lossy to identify the position of the integrand poles. It also indicates that the superfluid cannot be truly lossless to an ac field. Parts (a) and (b) of Fig. 1 are the plots of the complex dielectric constant for niobium as a function of frequency for different temperatures. As can be seen from the figure, the imaginary part of the dielectric constant is not equal to zero for the case $\omega \neq 0$, indicating loss in the material because some electrons are not in the superconducting state and can be scattered by phonons and impurities. This suggests that we cannot expect a superconductive cavity, once excited, to oscillate forever.

In the above discussion, we have assumed that the current of normal electrons and the electric field obey a local relation. Otherwise, the current density at a point is determined by an integral involving the electric fields up to about a coherence length from that point. This anomalous skin effect has to be dealt with by a nonlocal theory [13]. We have also assumed extremely low fields, where the nonlinearity of the superconductor can be neglected.

# III. The London Equations

The hydrodynamic model of a superconductor and its combination with one of Maxwell's equations lead to the well-known London equations. Consider a superconductor at a very low temperature so that only the superfluid needs to be considered. The combination of (1) and (4)

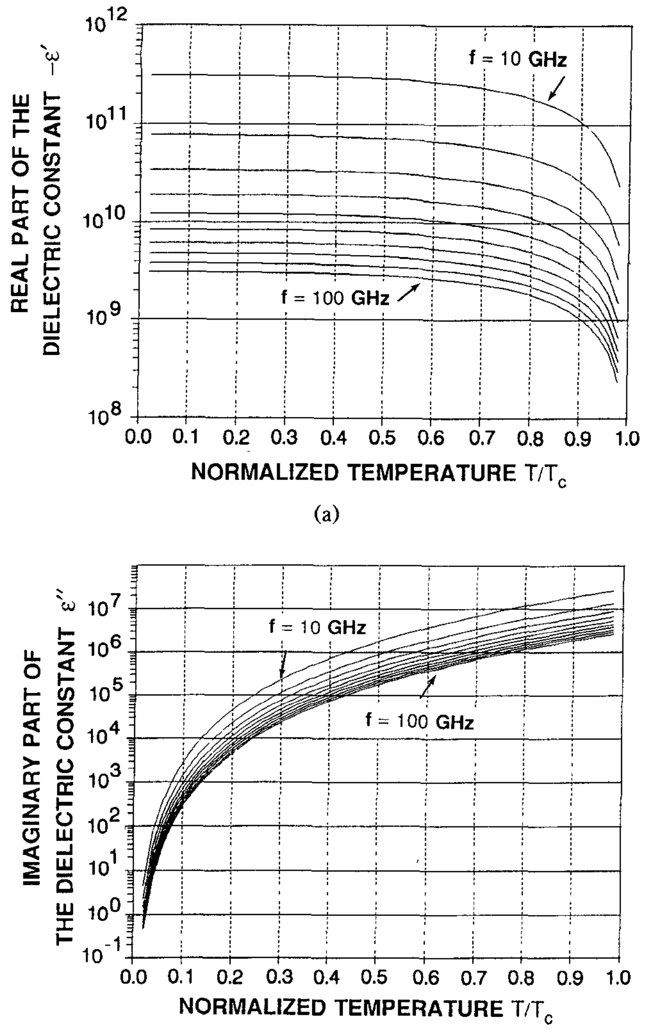


Fig. 1. (a) Calculated real part of the complex dielectric constant of niobium $-\epsilon'$ as a function of the normalized temperature for various frequencies (10 GHz to 100 GHz at a 10 GHz step) based on the two-fluid model. (b) Calculated imaginary part of the complex dielectric constant of niobium $\epsilon''$ as a function of the normalized temperature for various frequencies (10 GHz to 100 GHz at a 10 GHz step) based on the two-fluid model.

yields the first London equation,

$$\Lambda \frac{\partial \vec{J}_s}{\partial t} = \vec{E} \tag{14}$$

where $\Lambda = m/ne^2$. Substituting (14) into Faraday's law,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{15}$$

we have the following relation:

$$\frac{\partial}{\partial t}\left[\Lambda \nabla \times \vec{J}_c + \vec{B}\right] = 0. \tag{16}$$

Excluding dc, we obtain the second London equation,

$$\Lambda \nabla \times \vec{J}_s + \vec{B} = 0. \tag{17}$$

It is noted that London derived the above equation using general arguments of quantum mechanics, so the dc case does not have to be excluded. Substituting (17) into the other Maxwell equation,

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}_s \tag{18}$$

and neglecting the displacement current in (18), we obtain

$$\nabla \times \nabla \times \vec{B} = -\frac{\mu_0}{\Lambda}\vec{B} \tag{19}$$

which is the equation for the magnetic field inside a superconducting material. Equation (19) predicts that magnetic fields diminish exponentially in the superconductor. The displacement current is negligible in (18) at microwave frequencies. Equation (19) can be solved to find the penetration depth of the magnetic fields into a superconductor, which is typically 100 nm.

## IV. The Meissner Effect

A superconductor is often considered to be a lossless conductor at $T = 0$. Whether there is any difference between a superconductor and a perfect conductor was not an issue until the Meissner experiment was performed [14]. It is normally accepted in the superconductor community that classical field theory alone fails to explain the Meissner effect, that is, the experimental fact that dc magnetic flux is displaced from the interior of a superconductor.

### A. Static Approach

Equation (19) supports the Meissner effect in that at a minute distance from the surface, the magnetic fields are practically zero. It is a well-known fact that a dc magnetic field readily passes through a piece of conductor such as silver, copper, or any other nonmagnetic material. The classical electrostatic theory for the dc case states that

$$\nabla \times \vec{E} = 0 \tag{20}$$

and from (15) it follows that

$$\frac{\partial \vec{B}}{\partial t} = 0. \tag{21}$$

Therefore, the magnetic fields are "locked in" as the material is cooled into the superconducting state, contradicting the Meissner effect. Unfortunately, when we derived (17), from which (19) was obtained, we excluded dc. Consequently, it is not proper to use it to support the Meissner effect without considering the history of the system.

### B. Dynamic Approach

To reconcile classical field theory with Meissner's experiment, we need to consider dc as a limiting case of dynamics. In fact, strictly speaking, Meissner's experiment was not done at dc. In the experiment, a superconductor specimen was placed in a dc magnetic field at room temperature. The material was then cooled until it reached the superconducting state. Since, in the process of cooling, the dielectric parameter or the conductivity of the material changed with time, the interior fields, both electric and magnetic, were functions of time, even though the applied field was dc.

From the dynamic point of view, it is not difficult to visualize that as a material becomes superconducting, the tangential components of the electric field on the surface must gradually vanish and it follows that the normal component of the magnetic flux density also must vanish. The quasi-static solution to this problem results in separate solutions for the magnetic and electric fields. In order to satisfy the continuity condition for the normal component of the magnetic flux density, the permeability of the material must approach zero, i.e., $\mu \to 0$. In order to satisfy the vanishing tangential electric field, the corresponding boundary condition is $|\epsilon| \to \infty$.

As $\mu \to 0$, the magnetic flux density in the material must vanish, which is consistent with Meissner's experiment. And, $\lim_{\omega \to 0} |\epsilon_r(\omega)| \to \infty$ is also consistent with the previous discussions of the material properties of superconductors. Thus, Meissner's experiment should be viewed through its time history instead of as a strictly dc event. In that case, classical electrodynamic theory will be consistent with the Meissner effect. The above qualitative discussion of the Meissner effect does not yield a penetration depth for the fields, because we have assumed the tangential electric field on a superconductive surface to be exactly zero. In reality, there is a minute residual field inside the superconductor, as shown by the solution to the London equation. This becomes important in samples with dimensions of the order of the penetration depth.

The above quasi-static approach was used by Bethe in small-hole theory in microwave coupling [15]. In that case the magnetic field surrounding a perfectly conducting magnetic material was to be calculated. Bethe's quasi-static field results in $\epsilon \to 0$ and $\mu \to \infty$, which is the dual case of the above discussion. There is already a considerable amount of theoretical and experimental evidence for the validity of Bethe's theory.

## V. What is a Superconductor?

A material is usually classified as a dielectric, a conductor, or a semiconductor. The name superconductor immediately implies that it belongs to the conductor group. Similarly, the name dielectric immediately suggests an insulator. Actually, in the extreme case of a dielectric material with $\epsilon_r \to \pm\infty$, the distinction between a dielectric and a conductor is not so clear.

Consider the Maxwell equation

$$\nabla \times \vec{H} = (j\epsilon_0\epsilon_r + \sigma)\vec{E}. \tag{22}$$

When $\sigma \to \infty$ and $\vec{E} \to 0$, this implies a perfect conductor. The continuity of the tangential component of the electric field, $E_t$, leads to $E_t = 0$ on the surface. Indeed, $\epsilon_r \to \pm\infty$ also leads to the same boundary condition. Therefore, under ideal conditions, a perfect conductor ($\sigma \to \infty$) is not distinguishable from a perfect hyperdielectric material ($\epsilon_r \to \pm\infty$).

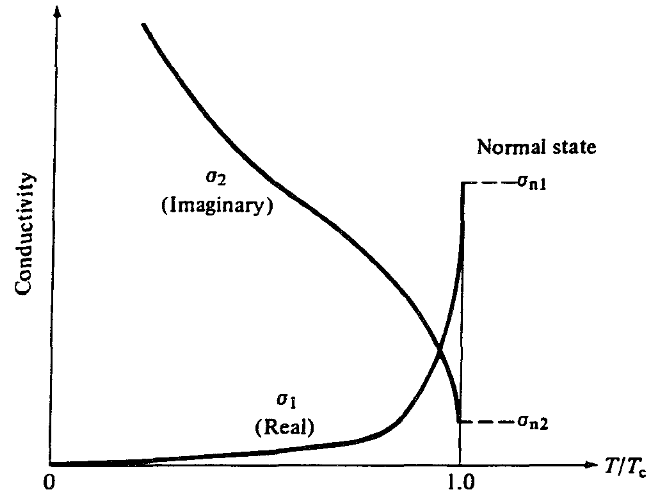


Fig. 2. Real and imaginary components of the conductivity $\sigma = \sigma_1 - j\sigma_2$ as a function of temperature (after [8]).

Accepting (8) as a legitimate formula for the dielectric constant of a superconductor, one immediately concludes that for $\omega < \omega_s$, a superconductor is actually a negative dielectric material. Since $\omega_s$ is of the order of 1–10 THz for most superconductors, we can safely assume that superconductors are negative dielectric materials, especially at dc. Fig. 2 shows a graph of the conductivity versus temperature of a superconductor at nonzero frequency, which has been used frequently in the literature. The conductivity is shown to approach infinity along the negative imaginary axis as the temperature approaches absolute zero. Therefore, a superconductor can be regarded as a material with negative imaginary conductivity or one with a negative real dielectric constant. Since an imaginary value for a material constant loses its meaning at dc and since many experiments on superconductors have been done at dc, classifying superconductors as negative dielectric materials does have its legitimacy and will simplify the computation of superconductive boundary problems.

## VI. Electromagnetics of Negative Dielectrics

Once a superconductor is identified as a negative dielectric material, many of the myths surrounding these materials are dispelled. One can just solve Maxwell's equation as with any other penetrable material. Of course, the dielectric constant has to be properly evaluated. Conventional perturbation methods are applicable to a superconductor, provided that the surface impedance (or admittance) can be determined. Some basic properties of negative dielectrics are now discussed.

### A. Energy Density in Negative Dielectrics

As we know from basic electromagnetic theory, the energy density in an electric field can be expressed as

$$W_E = \frac{1}{2}\epsilon_0\epsilon_r E(t)^2. \tag{23}$$

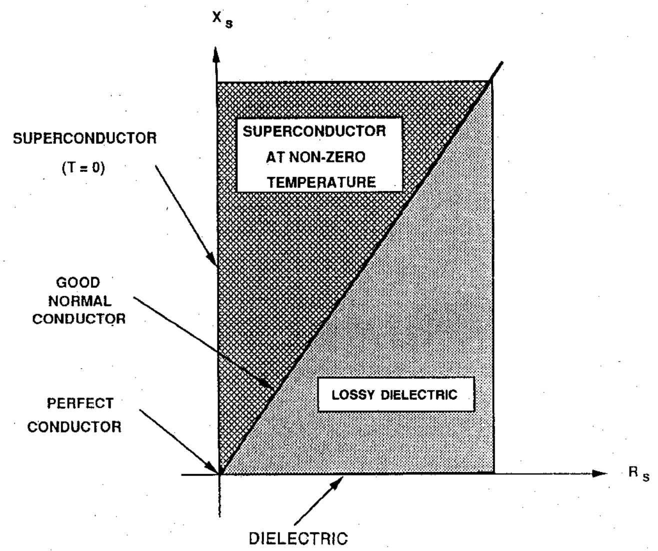


Fig. 3. Complex surface impedance for dielectric, conductors, and superconductors.

For a harmonic field, the time average energy density is

$$\overline{W_E}=\frac{1}{4}\epsilon_0\epsilon_r\overline{E}\cdot\overline{E}. \quad (24)$$

For superconductor or negative dielectrics, the dielectric constant is complex and frequency dependent: $\epsilon(\omega)=\epsilon'(\omega)-j\epsilon''(\omega)$ with $\epsilon'(\omega)<0$. In the case of high-$T_c$ materials, dielectric constants are even anisotropic. Under those conditions, the above formula is no longer valid. For a dispersive medium, we have to consider a wave packet instead of a "single" frequency, which results in the following equation for a source-free, lossless medium [16]:

$$\langle W\rangle=\frac{1}{4}\left[\vec{E}^*\cdot\left(\frac{\partial\omega\epsilon'}{\partial\omega}\right)_0\vec{E}+\mu_0\vec{H}^*\cdot\vec{H}\right] \quad (25)$$

where $\langle W\rangle$ is the time average energy density. For lossless, anisotropic, dispersive dielectrics, we have the following expression for the electric energy density [17]:

$$\langle W_e\rangle=\frac{1}{4}\vec{E}^*\cdot\frac{\partial\omega\overleftrightarrow{\epsilon}}{\partial\omega}\cdot\vec{E} \quad (26)$$

where the dielectric constant is a dyadic. The electric energy stored in a dielectric system is potential energy while the magnetic energy is kinetic. For a negative-dielectric-constant medium, the electric energy becomes kinetic and is associated with the motion of the paired electrons.

### B. Surface Impedance

The surface impedance of a dielectric, a normal metal, and a superconductor (negative dielectric) are shown in Fig. 3. All these materials have their surface impedances in the first quadrant of the complex plane. The surface impedance of a (positive) lossless dielectric material lies on the real axis, a good conductor is along a 45° line, and a superconductor (negative dielectric) is close to or along the positive imaginary axis, depending upon the operation temperature. As the dielectric constant approaches infinity or the conductivity approaches infinity, the surface impedances approach zero along their respective loci. When they approach the origin, we cannot distinguish a superconductor from a perfect conductor macroscopically. However, in reality, even in the superconducting state, the surface impedance of a superconductor does not reach zero. Instead, it stays at a point on the imaginary axis close to the origin, as indicated by the London equations. The surface inductive reactance of a good conductor is the same as its resistance, but the reactance of a superconductor is much larger than its resistance. It is this reactance that makes the electric and magnetic fields almost 90° out of phase, so the loss is quite small. The surface impedance of a superconductor can be obtained from its conductivity,

$$Z_s=\sqrt{\frac{\mu}{\epsilon}}=\sqrt{\frac{j\omega\mu_0}{\sigma}}=\sqrt{j\omega\mu_0\frac{\sigma_1+j\sigma_2}{\sigma_1^2+\sigma_2^2}}$$
$$\approx\sqrt{\frac{\omega\mu_0}{\sigma_2}}\left(\frac{1}{2}\frac{\sigma_1}{\sigma_2}+j\right) \quad (27)$$

where $\sigma=\sigma_1-j\sigma_2$ is the complex conductivity of a superconductor, with $\sigma_2>0$ and $\sigma_1\ll\sigma_2$. From the two-fluid model, $\sigma_1$ and $\sigma_2$ can be expressed as

$$\sigma_1=\frac{n_ne^2\tau_n}{m(1+\omega^2\tau_n^2)} \qquad \sigma_2=\frac{n_se^2}{m\omega}+\frac{n_ne^2(\omega\tau_n)^2}{m\omega(1+\omega^2\tau_n^2)}. \quad (28)$$

Since $\omega\tau_n$ is small, and $n_n\ll n_s$ for a superconductor at temperature far below $T_c$, we expect $\sigma_1/\sigma_2$ to be very small. That means that the surface resistance is very small. At 10 GHz, the microwave surface resistance of a YBCO film on $LaAlO_3$ substrate has been found to be as low as 20–50 $\mu\Omega$ at 4.2 K and 300–400 $\mu\Omega$ at 77 K. These surface resistance values are lower than those for Cu at 4.2 K and 77 K by factors of 120 and 15, respectively [18]–[21]. Such a small surface resistance gives a small power loss,

$$W_L=\frac{1}{2}\operatorname{Re}(Z_s)|\vec{J}|^2. \quad (29)$$

One might expect that superconductors will have a great impact on microwave and millimeter-wave technology, especially on devices and circuits where losses have been a major performance limitation, such as small antennas, phase shifters, filters, resonators, delay lines, and some analog signal processing circuits [22]–[31].

### C. Surface Wave

A surface wave is a wave that travels along a surface without a lessening of its total time average Poynting vector integrated over a large closed loop, i.e.,

$$W=\oint_c\vec{P}\cdot\hat{\rho}\,dc \quad (30)$$

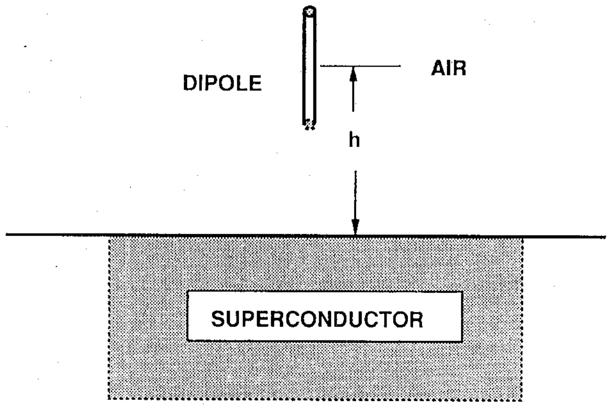


Fig. 4. A dipole with a distance $h$ above superconductor plane.

where $\vec{P}=\frac{1}{2}\mathrm{Re}[\vec{E}\times\vec{H}^*]$, $c$ is a large closed loop on the surface, and $\hat{\rho}$ is a unit radial vector tangent to the surface and normal to $c$. For a surface wave, the power, $W$, should be independent of the radius of the closed loop, $c$. That is to say, as the integral loop extends to infinity ($\rho\to\infty$), $|\vec{P}|\to A/\rho$, i.e., $|\vec{E}|$ or $Z_0|\vec{H}|\to B/\sqrt{\rho}$, where $A$ and $B$ are two constants. Normally, a surface wave can be created between two planar surfaces of total reflection, such as a dielectric layer or a substrate with a conducting ground, and the physics of such a surface wave is well understood [32], [33]. However, Sommerfeld, in his treatment of radiation over a lossy ground, discovered that a surface wave could also be obtained with only a single dielectric interface. The configuration of Sommerfeld's analysis is depicted in Fig. 4. The potentials of the electromagnetic fields, $\Pi$, are represented by Fourier–Bessel integrals, known as Sommerfeld's integrals [33]. Parts of these integrals may be represented by residual integrals, which result in

$$\Pi_0=2\pi j\frac{k_{\mathrm{sup}}^2}{\kappa}H_0^{(2)}(pr)e^{-\sqrt{p^2-k_0^2}z},\qquad z\geqslant 0 \quad (31)$$

$$\Pi_{\mathrm{sup}}=2\pi j\frac{k_0^2}{\kappa}H_0^{(2)}(pr)e^{-\sqrt{p^2-k_{\mathrm{sup}}^2}z},\qquad z\leqslant 0 \quad (32)$$

where $H_0^{(2)}$ is the cylindrical Hankel's function of the second kind and $\kappa$ is a known constant. Also, $k_{\mathrm{sup}}$ is the wavenumber in the superconductor in the lower half-space, and $p$ is the pole of the integrand of the Sommerfeld integrals:

$$\frac{1}{p^2}=\frac{1}{k_{\mathrm{sup}}^2}+\frac{1}{k_0^2} \quad (33)$$

or

$$p^2=\frac{k_0^2k_{\mathrm{sup}}^2}{k_0^2+k_{\mathrm{sup}}^2}. \quad (34)$$

If $|k_{\mathrm{sup}}|\gg k_0$, we may approximate $p$ by

$$p=k_0\left[1-\frac{1}{2}\frac{k_0^2}{k_{\mathrm{sup}}^2}\right]. \quad (35)$$

The above formulas were derived by Sommerfeld. The asymptotic forms of (31) and (32), as $|pr|\to\infty$, are as follows:

$$\Pi_0=2\sqrt{\frac{2\pi j}{pr}}\frac{k_{\mathrm{sup}}^2}{\kappa}e^{-jpr-\sqrt{p^2-k_0^2}z},\qquad z\geqslant 0 \quad (36)$$

$$\Pi_{\mathrm{sup}}=2\sqrt{\frac{2\pi j}{pr}}\frac{k_0^2}{\kappa}e^{-jpr+\sqrt{p^2-k_{\mathrm{sup}}^2}z},\qquad z\leqslant 0. \quad (37)$$

At the $z=0$ plane, the potentials satisfy the conditions of a surface wave created by a localized source. If we inspect (35) closely, we find that if the lower half-space were an ordinary dielectric material, which has a positive dielectric constant, then $p<k_0$. Thus, the waves in (36) and (37) would be fast waves, which would radiate and could not become a surface wave. However, the waves can be retarded by losses in the lower half-space. That is why the Sommerfeld type of surface wave can only be excited if there is loss in the medium. The situation is quite different for a negative dielectric material or a superconductor. Since in that case $k_{\mathrm{sup}}^2<0$ and $p>k_0$, the waves in (36) and (37) are naturally slow, and there is a natural surface wave. However, for a superconducting state $|k_{\mathrm{sup}}^2|\gg k_0^2$, $p$ is practically equal to $k_0$; therefore the surface wave, if excited, would not be stable. One method of getting a stable surface wave is to operate the system at a temperature near but below $T_c$. Even though in that region the loss may increase, it may still be much less than conventional conduction loss.

## VII. Current Densities

Traditionally, Maxwell's equations are studied in the frequency domain, where a dispersive dielectric medium is easily represented by the equation

$$\nabla\times\vec{H}=j\omega\epsilon(\omega)\vec{E}. \quad (38)$$

However, the real and imaginary parts of $\epsilon(\omega)$ must be related by the Kramers–Kronig relations, (11) and (12). The Kramers–Kronig relations were derived from the causality relation between the flux density, $\vec{D}$, and $\vec{E}$ in the time domain,

$$\vec{D}(t)=\int_0^\infty G(\tau)\vec{E}(t-\tau)\,d\tau \quad (39)$$

where $G(\tau)\equiv 0$ for $\tau<0$. The Fourier transform of such a function results in the Kramers–Kronig relations of (11) and (12).

It is of interest to obtain some physical insight by relating $\epsilon(\omega)$ and $G(\tau)$. The time-domain representation of (38) is

$$\nabla\times\vec{H}=\frac{\partial}{\partial t}\int_0^\infty G(\tau)E(t-\tau)\,d\tau=\int_0^\infty\frac{\partial G(\tau)}{\partial\tau}E(t-\tau)\,d\tau. \quad (40)$$

The first integral of (40) represents the electric flux density; the second, obtained using integration by parts, rep-

TABLE I
*Current Densities in the Time and Frequency Domains*

| Current Type | Expression in FD | $G(t)$ in TD | $\frac{\partial G(t)}{\partial t}$ in TD |
|---|---|---|---|
| Displacement | $j\omega\epsilon_0\vec{E}$ | $\epsilon_0\delta(t)$ | |
| Conduction | $\sigma\vec{E}$ | | $\sigma\delta(t)$ |
| Relaxation | $\frac{\omega_n^2}{(j\omega+\gamma_n)}\vec{E}$ | | $\omega_n^2 e^{-\gamma_n t}u(t)$ |
| Super | $\frac{\omega_s^2}{j\omega}\vec{E}$ | | $\omega_s^2 u(t)$ |

resents the total current density. Consider the time-domain Maxwell's equation

$$\nabla\times\vec{H}=j\omega\vec{D}+\vec{J}. \tag{41}$$

The first term of the right-hand side of (41) is the displacement current, where $\vec{D}=\epsilon_0\vec{E}$. The current density, $\vec{J}$, may occur in different forms in different cases. For conventional conduction current, $\vec{J}=\vec{J_c}=\sigma\vec{E}$, and in a superconductor, $\vec{J}=\vec{J_n}+\vec{J_s}$, where

$$\vec{J_n}=\frac{\epsilon_0\omega_n^2\tau_n}{j\omega\tau_n+1}\vec{E} \quad\text{and}\quad \vec{J_s}=\frac{\epsilon_0\omega_s^2}{j\omega}\vec{E}.$$

The corresponding functions $G(t)$ and $\partial G/\partial t$ of each current are listed in Table I.

It is obvious that the terms $G(t)=\epsilon\delta(t)$ and $\partial G/\partial t=\sigma\delta(t)$ correspond to the displacement and conduction current, respectively. In those cases, the flux densities and current response to the electric field are instantaneous and they disappear as soon as the electric field disappears. The normal current corresponds to an exponentially decaying $\partial G/\partial t$ term; i.e., the current depends not only on the instantaneous value of the field but also on the history of the field. Hence, it does not obey Ohms law. In view of the possible confusion between the normal current and the ohmic conduction current, we have renamed the normal current the relaxation current in the table. This term contains inertial effects while the conventional conduction current does not. The supercurrent is the limiting case of the "relaxation current" when $\gamma_n(=1/\tau_n)\to 0$, i.e., the case when the relaxation time becomes infinite. We may consider the conduction current and supercurrent as two limiting cases of the "relaxation current," the conduction current for $\omega\to 0$ and the supercurrent for $\gamma_n\to 0$. In the former, the relaxation time tends to zero and is collision dominated $(\omega_n^2/(j\omega+\gamma_n)\to\omega_n^2/\gamma_n=ne^2/m\gamma_n=\sigma)$. In the latter, the relaxation time is infinite and is purely inertial. They would respond very differently to electromagnetic fields, a fact that has also been discussed by Pippard [34].

The above limiting cases lead us to believe that the "perfect conductor" cannot exist except at dc since $\gamma_n$ must be much larger than $\omega$ for the "relaxation current" to become conduction current.

## VIII. Computational Considerations

In Section IV, we have mentioned that the solutions of electromagnetic boundary value problems are not any more complicated for a superconductor than for any other penetrable medium, once it is treated as a negative dielectric material. While that statement is obviously true in the frequency domain, it is not so obvious in the time domain, because a superconductive medium is dispersive. The complication in the time-domain computation involving a superconductor arises from the necessity of computing the convolution integral,

$$D(t)=c\int_0^\infty e^{-\gamma_n\tau}E(t-\tau)\,d\tau \tag{42}$$

where $c$ is a constant. In reality, the integral of (42) can be calculated without much overhead in storage for past electric fields. A change of variables leads to

$$\vec{D}(t)=ce^{-\gamma_n t}\int_t^{-\infty}e^{\gamma_n\xi}\vec{E}(\xi)\,d\xi=-ce^{-\gamma_n t}\int_{-\infty}^{t}e^{\gamma_n\xi}\vec{E}(\xi)\,d\xi. \tag{43}$$

The integral in (43) can be "accumulated" in the process of time-domain computation. Since $\gamma_n$ is a positive number, the multiplier $e^{-\gamma_n t}$ becomes smaller and the integration term gets larger as $t$ increases. Therefore (43) is not a stable form for computation. In the numerical integration of (43), however, a little manipulation results in

$$D(n\Delta t)=D[(n-1)\Delta t]e^{-\gamma_n\Delta t}-cE(n\Delta t)\,\Delta t \tag{44}$$

which gives a very stable recursive formula. $D(n\Delta t)$ is the field at the $n$th time step. Since a superconducting medium described by the two-fluid model has the form of (42), its time-domain computation is quite simple even though the medium is dispersive.

## IX. Conclusion

This paper presents an understanding of superconductive materials in the context of classical electrodynamics. The basics of its electromagnetic properties are presented and a possible surface wave phenomenon is suggested. The relationships between current densities and electric fields are depicted in the time domain. A new interpretation of the Meissner effect has been presented. The phrases "negative dielectric" and "relaxation current" may sound controversial and unnecessary. However, they are introduced to avoid confusion for most electromagnetic engineers, who have a preconceived understanding of the name "conductor" and the implications of the name "normal conductor." The electromagnetic interpretation of the Meissner effect is an effort to remove the mystery associated with superconductors, which, somehow, seem to defy the logic of classical electromagnetic theory. Although the physics of superconductors are studied at the quantum level, the macroscopic properties of the material from which it is derived must be consistent with the classical theory of electromagnetics. We believe the discussions presented in this paper will be helpful to electrical engineers involved with the applications of superconductors. Solving classical electrodynamic boundary value problems will be an unavoidable step in the eventual applications of these materials in microwave and millimeter-wave devices and electronics.

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